MATHEMATICS

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AIM POINT
MATHEMATICS
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THREE DIMENSIONAL GEOMETRY

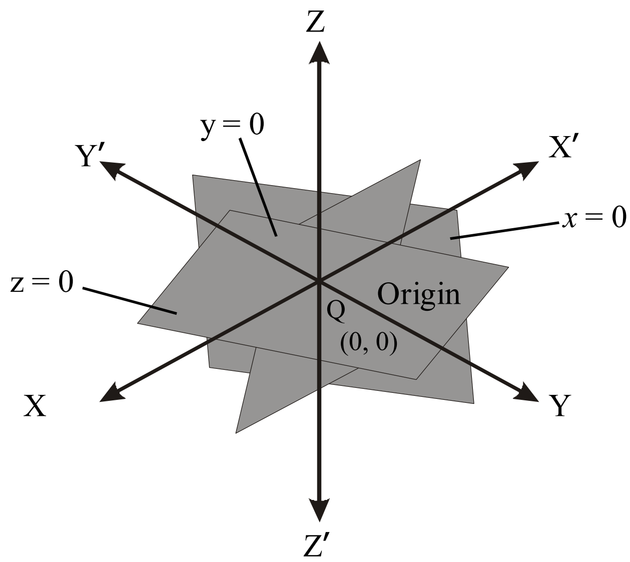
& Their Properties

CONTENTS

THINGS TO REMEMBER

✷ Cartesian Coordinates in Three Dimensions

Let *X′OY, Y′OY* and *Z′OZ* be three mutually perpendicular lines intersecting at O. O is the origin and line *X′OX, Y′OY* and *ZOZ′* are called *x*-axis, y-axis and z-axis respectively. These three lines are also called the rectangular axes of coordinates The planes containing these three lines in pairs, determine three mutually perpendicular planes *XOY, YOZ* and *ZOX*.



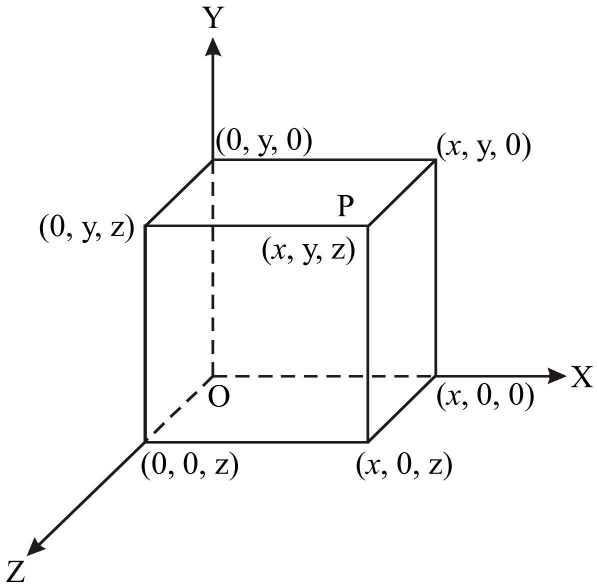
The three planes divide space into eight cells called octants.

The following table show the sign of coordinates of poits in various octants.

Octant Coordinate	*x*	y	z
OXYZ	+	+	+
OX'YZ	–	+	+
OXY'Z	+	–	+
OXYZ'	+	+	–
OX'Y'Z	–	–	+
OX'YZ'	–	+	–
OXY'Z'	+	–	–
OX'Y'Z'	–	–	–

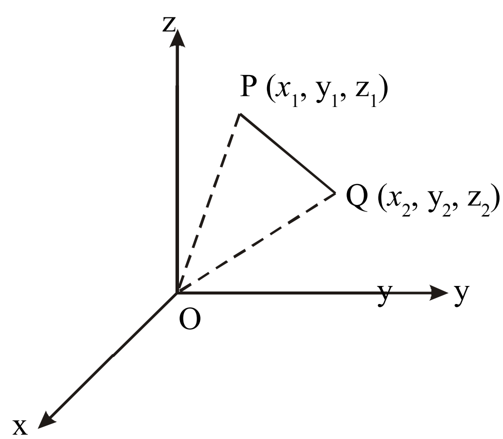
The cartesian coordinates (*x*, y, z) of a point P in a space are the number at which the planes through P perpendicular to the axes out the axes. The coordinates of a point on x-axis are (*x*, 0, 0), on y-axis are (0, y, 0) and on z-axis (0, 0, z).

The standard equations of *x*y-plane, yz-plane and z*x*-plane are z = 0, *x* = 0 and y = 0 respectively.



✷ Distance between Two Points

The distance between two points P(x_2, y_2, z_2) and Q(x_2, y_2, z_2) is given by



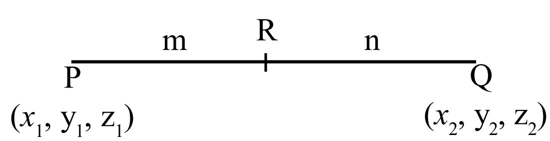
$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

✷ Section Formulae

1. Internal Division

Let P(x_1, y_1, z_1) and Q(x_2, y_2, z_2) be two points. Let R be a point on the line segment joining P and Q internally in the ratio m : n Then, the coordinates of R are

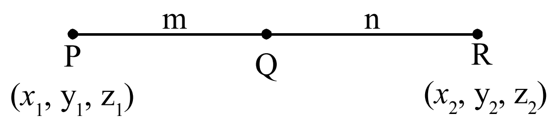
$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n}\right)$$



2. External Division

If P and Q are such that R divides the join of P and Q externally in the ratio m : n. Then, the coordinates of R are

$$\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}, \frac{mz_2 - nz_1}{m-n}\right)$$



✷ Centroid of a Tetrahedron

Let A(x_1, y_1, z_1), B(x_2, y_2, z_2), C(x_3, y_3, z_3) and D(x_4, y_4, z_4) be the vertices of a tetrahedron, then centroid (G) of tetrahedron ABCD is

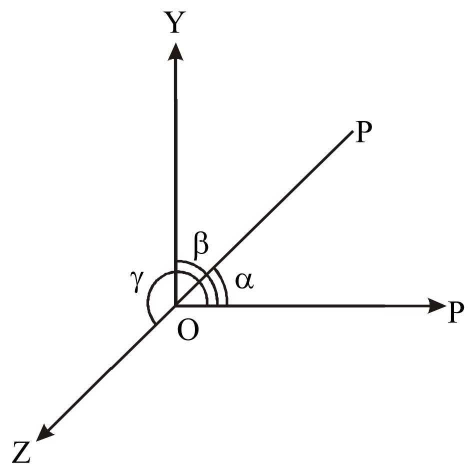
$$\left[\frac{1}{4}(x_1 + x_2 + x_3 + x_4), \frac{1}{4}(y_1 + y_2 + y_3 + y_4), \frac{1}{4}(z_1 + z_2 + z_3 + z_4)\right]$$

✷ Direction Cosines and Direction Ratios

Direction Cosines

If α, β, γ are the angle which a directed line segment *OP* makes with the positive directions of the coordinate axes *OX, OY, OZ* respectively, then cos α, cos β, cos γ, are known as the direction cosines of *OP* and are generally denoted by the letters l, m, n respectively, ie,

$$l = \cos\alpha,\ m = \cos\beta,\ n = \cos\gamma$$



Properties of Direction Cosine

1. If OP is a directed line segment with direction cosines l, m, n such that OP = r. Then, the coordinates of are (lr, mr, nr).
2. Sum of squares of direction cosine are always unity ie, $l^2 + m^2 + n^2 = 1$.
3. Parallel lines have same direction cosines.
4. Direction cosines of a line are always unique.
5. $0 < \alpha, \beta, \gamma < \pi$.

Direction Ratio

Let l, m, n be direction cosines of a line and a, b, c be three numbers such that $\frac{l}{a}=\frac{m}{b}=\frac{n}{c}$. Then direction ratios of the line are proportional to a, b, c.

Relation between Direction Cosines and Direction Ratios

If the direction ratios of a line are proportional to a, b, c then its direction cosine are

$$l=\pm\frac{a}{\sqrt{a^2+b^2+c^2}},\ m=\pm\frac{b}{\sqrt{a^2+b^2+c^2}},\ n=\pm\frac{c}{\sqrt{a^2+b^2+c^2}}$$

✱ **Angle between Two Lines**

If two lines whose direction cosine are (l_1, m_1, n_1) and (l_2, m_2, n_2), then angle θ between them is given by

cost =

If direction ratios are given (a_1, b_1, c_1) and (a_2, b_2, c_2) respectively, then

Now, if $l_1l_2 + m_1m_2 + n_1n_1 = 0$, lines are perpendicular and if $l_1 = l_2$, $m_1 = m_2$ and $n_1 = n_2$, lines are parallel.

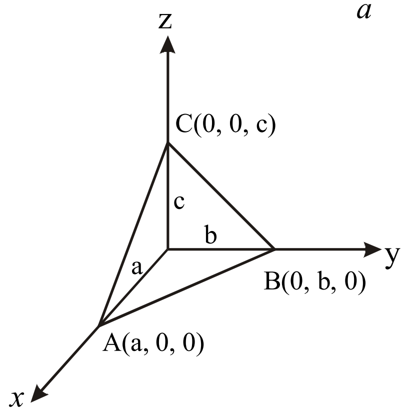
Similarly, If $a_1a_2 + b_1b_2 + c_1c_2 = 0$, then lines are perpendicular and if $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$, then lines are parallel.

✱ **Plane**

A plane is a surface such that, if any two point are taken on it, the line segment joining them lies completely on the surface.

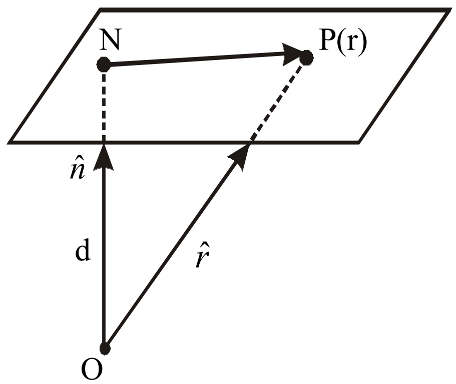
Equation of Plane in Different Form

1. **General Equation of a Plane** General eqution of a plane is ax + by + cz + d = 0.
2. **Equation of a Plane Passing Thorugh a Given Point** The general equation of a plane passing through a given point (x_1, y_1, z_1) is $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$, where a, b, c are direction ratios of a line perpendicular to plane.
3. **Intercept Form of the Equation of a Plane** The equation of a plane whose intercepts are a, b, c on the x-axis, y-axis and z-axis respectively, is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$.



4. **Equation of Plane in Normal Form** The equation of a plane in normal form is $lx + my + nz = p$ where l, m, and n are direction cosines of a line normal to plan and p is perpendicular distanec of the plane from origin.

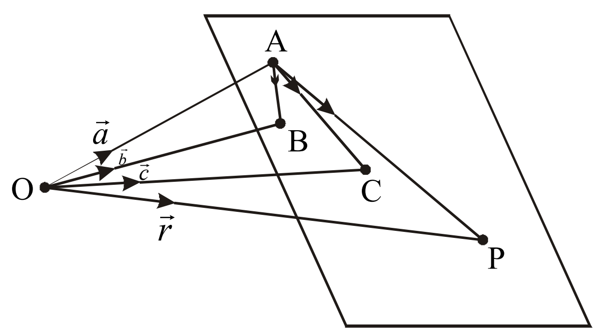
The vector equation of a plane normal to unit vector $\hat{n}$ at a distance d from the origin is $\hat{r} \cdot \hat{n} = d$.



5. **Equation of a Plane passing Through Three Non-collinear** Points Equation of a plane passing through the three non-collinear point $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$ is.

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

Let A, B and C be the three given points with position vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ respectively and P be a point in the plane with position vector $\vec{r}$.



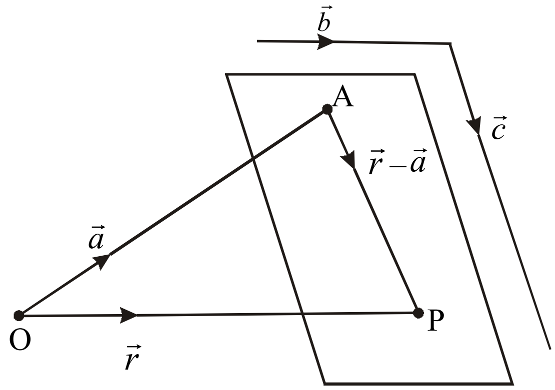
Then, equation of plane is

$$[\vec{r} - \vec{a} \;\; \vec{b} - \vec{a} \;\; \vec{c} - \vec{a}] = 0$$

or $$[\vec{r}\,\vec{b}\,\vec{c}] + [\vec{r}\,\vec{a}\,\vec{b}] + [\vec{r}\,\vec{c}\,\vec{a}] = [\vec{a}\,\vec{b}\,\vec{c}]$$

6. **Equation of Plane Passing Through a Point and Parallel to two Vectors** Equation of a plane passing through a point a whose position vector is $\vec{a}$ and parallel to two vectors $\vec{b}$ and $\vec{c}$ is

$$[\vec{r}\,\vec{b}\,\vec{c}] = [\vec{a}\,\vec{b}\,\vec{c}]$$



Angle between Two Planes

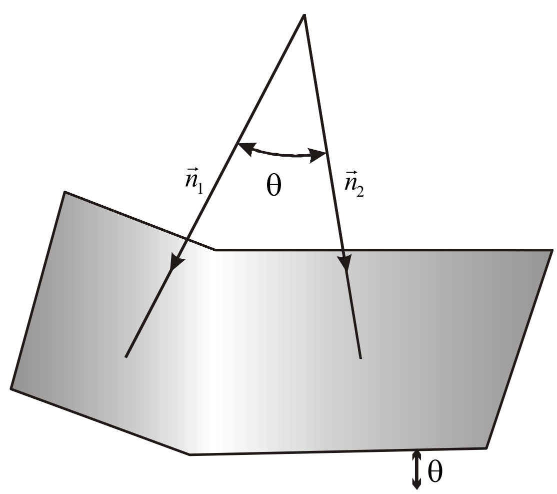
Angle between plane $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is given by

$$\cos\theta = \left| \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^1}\sqrt{a_2^2 + b_2^2 + c_2^2}} \right|$$

where θ is acute angle between the planes.

The angle between two intersecting planes is defined to be the (acute) angle detemined by their normal vectors.

Let θ be the angle between the plane $\vec{r} \cdot \vec{n}_1 = d_1$ and $\vec{r} \cdot \vec{n}_2 = d_2$ then



Now, two planes are perpendicular, if

$$\vec{n}_1 \cdot \vec{n}_2 = 0 \qquad \text{or} \qquad a_1a_2 + b_1b_2 + c_1c_2 = 0$$

and parallel, if $\dfrac{\vec{n}_1}{\vec{n}_2} = \lambda$ or $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$.

Distance of a Point from a Plane

The distance of a point $P(x_1, y_1, z_1)$ from a plane $ax + by + cz + d = 0$ is given by

$$\left| \frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}} \right|$$

The distance of a point P(a) from the plane $\vec{r} \cdot \vec{n} = q$ is given by

$$\left|\frac{q-\vec{a}\cdot\vec{n}}{|\hat{n}|}\right|$$

✱ Distance between Two Parallel Planes

The distance between two parallel planes $ax + by + cz + d_1 = 0$ and $ax + by + cz + d_2 = 0$ is given by

$$d=\left|\frac{d_2-d_1}{\sqrt{a^2+b^2+c^2}}\right|$$

Important Facts Related to Plane

1. Image of point in a plane Image (x, y, z) (or reflection) of a point (x_1, y_1, z_1) in a plane $ax + by + cz + d = 0$ is $\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}=\frac{-2(ax_1+by_1+cz_1+d)}{a^2+b^2+c^2}$.
2. Foot (x, y, z) of perpendicular drawn from a point (x_1, y_1, z_1) to the plane $ax + by + cz + d = 0$ is $\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}=-\frac{(ax_1+by_1+cz_1+d)}{a^2+b^2+c^2}$.
3. Any plane parallel to XY plane is z = constant, similarly plane parallel to YZ plane is x = constant and plane parallel to ZX plane is y = constant, $x = 0$, $y = 0$, and $z = 0$, are respectively YZ, ZX and XY planes.
4. Any plane parallel to X-axis is of the form $by + cz = d$
5. Position of the points Points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are on same side of plane $ax + by + cz + d = 0$ if $ax_1 + by_1 + cz_1 + d$ and $ax_2 + by_2 + cz_2 + d$ are of same sign. if they are of opposite sign, then the points are on the opposite sides.
6. Equation of plane parallel to planes $ax + by + cz + d_1 = 0$ and $ax + by + cz + d_2 = 0$ and equidistance from them is

$$ax+by+cz+\left(\frac{d_1+d_2}{2}\right)=0$$

7. If $\vec{a}$, $\vec{b}$, $\vec{c}$, $\vec{d}$ are coplanar, then

$$(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = 0$$

✱ Family of Planes

Equation of Family of Planes

1. Let $P_1 \equiv a_1x + b_1y + c_1z + d_1 = 0$ and $P_2 \equiv a_2x + b_2y + c_2z + d_2 = 0$ be two planes, then $P_1 + \lambda P_2 = 0$ (where λ is a parameter) represents family of planes passing through line of intersection of the planes $P_1 = 0$ and $P_2 = 0$. Let $S_1 \equiv \vec{r}\cdot\vec{n}_1 = q_1$ and $S_2 \equiv \vec{r}\cdot\vec{n}_2 = q_2$ be two planes, then $S_3 \equiv S_1 + \lambda S_2$ represents family of planes.
2. $ax + by + cz = k$ represents family of planes parallel to the plane $ax + by + cz + d = 0$. (where k is a parameter).

Equation of Planes Bisecting the Angle between Two Planes

Equation of the planes biscting the angle between the planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ are

$$\frac{a_1x+b_1y+c_1z+d_1}{\sqrt{a_1^2+b_1^2+c_1^2}} = \pm\frac{a_2x+b_2y+c_2z+d_2}{\sqrt{a_2^2+b_2^2+c_2^2}} \quad ...(i)$$

Bisector of Acute/Obtuse Angle

Write the equation of the given planes such that their constant terms (ie, d_1, d_2) ae positive.

(a) If $a_1a_2 + b_1b_2 + c_1c_2 > 0$, then origin lies in obtuse angle and hence, positive sign in (i) gives the bisector of the obtuse angle.

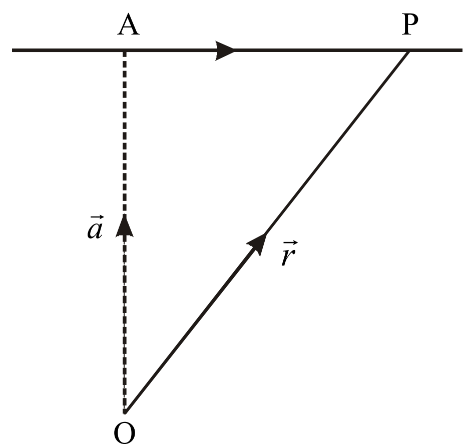
(b) If $a_1a_2 + b_1b_2 + c_1c_2 < 0$, then origin lies in acute angle and hence, positive sign in (i) gives the bisector of the acute angle.

✱ **Straidht Line**

Equation of Straight Line in Dfferent Forms

1. Equation of a Straight Line Passing Through a Given Point and Parallel to a Given Vector

Equation of straight line passing throught a point A with position vector a $(x_1\hat{i} + y_1\hat{j} + z_1\hat{k})$ and parallel to a vector b $(a\hat{i} + b\hat{j} + c\hat{k})$ is



$$\vec{r} = \vec{a} + \lambda\vec{b}$$

On putting the value of $\vec{r}$, $\vec{a}$ and $\vec{b}$, we get

$$x\hat{i} + y\hat{j} + z\hat{k} = (x_1\hat{i} + y_1\hat{j} + z_1\hat{k}) + \lambda(a\hat{i} + b\hat{j} + c\hat{k})$$

$\therefore \quad x - x_1 = \lambda a, \quad y - y_1 = \lambda b, \quad z - z_1 = \lambda c$

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

which is the required equation of line in cartesian form. Here a, b, c are direction ratio. if l, m, n are direction cosines, then equation of straight line is

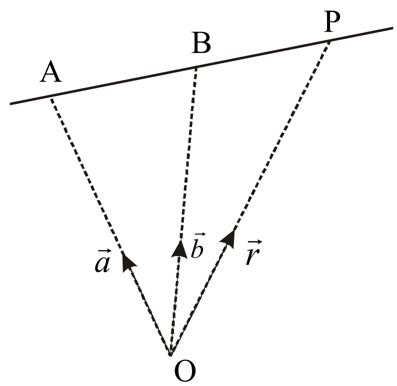
$$\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$$

This form of straight line is called symmetrical form.

2. Equation of Straight Line Passing Through Two Points

The equation of a line passing throught two points whose position vector are $\vec{a}$ and $\vec{b}$ is

$$\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$$



Equation of a straight line passing though (x_1, y_1, z_1) and (x_2, y_2, z_2)

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

Angle between the Two Lines

Let $\vec{r} = \vec{a}_1 + \lambda\,\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \lambda\,\vec{b}_2$ be equations of two straight lines. If t is the angle between them, then

$$\cos\theta = \frac{\vec{b}_1 \cdot \vec{b}_2}{|\vec{b}_1||\vec{b}_2|}$$

Also, If t the angle between

$$\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$$

and
$$\frac{x - x_2}{a_2} = \frac{y - y_2}{b_2} = \frac{z - z_2}{c_2}$$

then
$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

Condition of Perpendicularity

The lines are perpendicular, if

$$\vec{b}_1 . \vec{b}_2 = 0$$

or $a_1a_2 + b_1b_2 + c_1c_2 = 0$

Condition of Parallelism

The lines are parallel, if

$\vec{b}_1 = \lambda\vec{b}_2$, for same scalar λ

or $$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

Condition of Coplanarity of Two Lines

If the lines $\frac{x-x_1}{l_1}=\frac{y-y_1}{m_1}=\frac{z-z_1}{n_1}$ and $\frac{x-x_1}{l_2}=\frac{y-y_1}{m_2}=\frac{z-z_1}{n_2}$ are coplanar, then

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}=0$$

and equation of plane containing them is given by

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}=0$$

or $$\begin{vmatrix} x-x_2 & y-y_2 & z-z_2 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}=0$$

If the line $\vec{r}=\vec{a}_1+\vec{b}_1\lambda$ and $\vec{r}=\vec{a}_2+\lambda\vec{b}_2$ are coplanar then $(\vec{a}_2-\vec{a}_1).(\vec{b}_2-\vec{b}_1)=0$

✷ **<u>Skew Lines</u>**

Two non-parallel non-intersecting straight lines are caled skew lines.

Shortest Distance between Two Lines

Let the straight lines are $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$ d is shortest distance between them.

Then, $d=|(x_1-x_2)l+(y_1-y_2)m+(z_1-z_2)n|$,

where l, m, n are direction cosines of a line perpendicular to lines with (a_1, b_1, c_1) and (a_2, b_2, c_2).

So $$l\hat{i}+m\hat{j}+n\hat{k}=\frac{(\vec{a}\times\vec{b})}{|\vec{a}\times\vec{b}|}$$

Where $$\vec{a}=a_1\hat{i}+b_1\hat{j}+c_1\hat{k}$$

and $$\vec{b}=a_2\hat{i}+b_2\hat{j}+c_2\hat{k}$$

If are two skew lines, then the distance between them is $\left|\frac{(\vec{b}_1\times\vec{b}_2)(\vec{a}_2-\vec{a}_1)}{|\vec{b}_1\times\vec{b}_2|}\right|$ are these line intersect, if

$(\vec{b}_1 \times \vec{b}_2)(\vec{a}_2-\vec{a}_1)=0$ ie, shortest distance = 0

The distance between two skew lines $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$ is given by

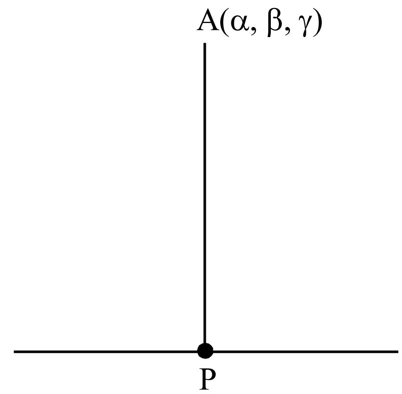
$$d=\frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{[(m_1n_2-m_2n_1)^2+(n_1l_2-n_2l_1)^2+(l_1m_2-l_2m_1)^2]}}$$

If $\vec{r}=\vec{a}_1+\vec{b}_1\lambda$ and $\vec{r}=\vec{a}_2+\mu\vec{b}_2$ are two parallel lines, then the distance between them is given by

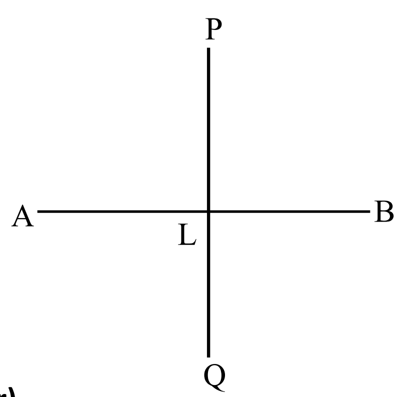
$$s=\frac{|(\vec{a}_2-\vec{a}_1)\times\vec{b}\,|}{|\vec{b}\,|}$$

Important Facts Related to Line

1. **Foot of Perpendicular form a Point A(α, β, γ) to the Line** $\frac{x-x_1}{l}=\frac{y-y_1}{m}=\frac{z-z_1}{n}$ If P be the foot perpendicular, then P is $(lr+x_1, mr+y_1, nr+z_1)$. Find the direction ratios of AP and apply the condition of perpendicular of AP and the given line. This will give the value of r and hence, the point P, which is foot of perpendicular.



2. **Length and Equation of Perpendicular** The length of the perpendicular is the distance AP and its equation is the line joining two known points A and P.
3. **Reflection of Image of a Point in a Straight Line** If the perpendicular PL form point P on the given line be produced to Q such that PL = QL, then Q is known as the image of reflection of P in the given line. Also, L is the foot of perpendicular or the projection of P on the line.



✱ Line and Plane

Equation of Plane Through a Given Line

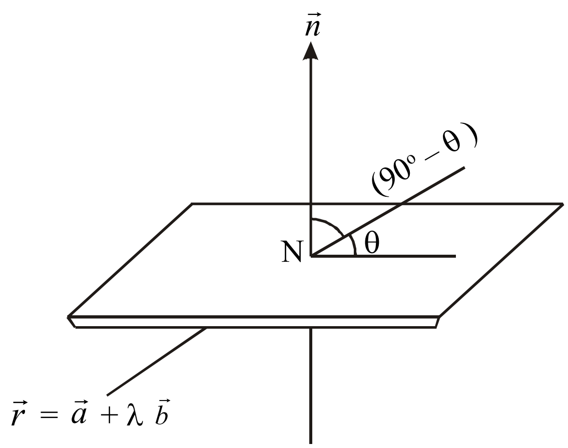
1. If equation of the line is given in symmetrical form as $\frac{x-x_1}{l}=\frac{y-y_1}{m}=\frac{z-z_1}{n}$ then equation of plane is $a(x+x_1)+b(y+y_1)+c(z+z_1)=0$ where a, b, c are given by al + bm + cn = 0.
2. If equation of line is given in general form as $a_1x+b_1y+c_1z+d_1=0=a_2x+b_2y+c_2z+d_2$, then the equation of plane passing throught this line is $(a_1x+b_1y+c_1z+d_1)+\lambda(a_2x+b_2y+c_2z+d_2)=0$.
3. If the plane pass thruogh parallel lines $\vec{r}=\vec{a}+\lambda\vec{b}$ and $\vec{r}=\vec{c}+\vec{b}$ then equation of the required plane is

$$[\vec{r}-\vec{a}\ \ \vec{c}-\vec{a}\ \ b]=0$$

Angle between a Line and a Plane

Let the equation of line be $\vec{r}=\vec{a}+\lambda\vec{b}$ and equation of plane is

$$\vec{r}\cdot\vec{n}=q$$



Let θ be the angle between a line and plane, then

$$\sin\theta=\frac{\vec{b}\cdot\vec{n}}{|\vec{b}||\vec{n}|}$$

Angle between a line $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$

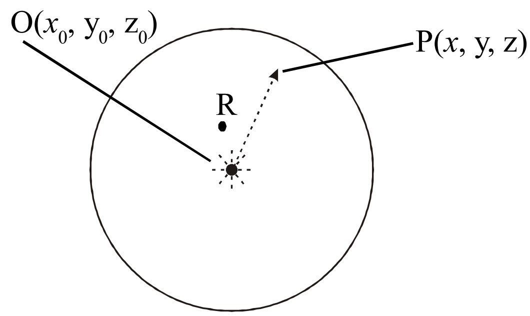
and plane $a_2x+b_2y+c_2z+d_2=0$ is given by

$$\sin\theta=\left|\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right|$$

✱ Sphere

A sphere is the locus of a point which moves in space such that its distane from a fixed point is always constant.

The fixed point is called the centre of the sphere and the fixed distance is called the radius of sphere.



Equation of Sphere in Different Form

1. If O (x_0, y_0, z_0) be the centre of sphere and radius of sphere is R, then equation of sphere is

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = R^2$$

Let C(α, β, γ) be te coordinates of centre with position vector c = $\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ and a be the radius then equation of sphere is | r – c | = a.

2. **General Equation of Sphere**

General equation of sphere is

$$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$$

with centre at (–u, –v, –w) and radius

$$R = \sqrt{u^2 + v^2 + w - d}$$

3. **Equation of Sphere in Diameter Form**

If P(x_1, y_1, z_1) and Q(x_2, y_2, z_2) are ends of diameter, then equation of sphere is

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) + (z - z_1)(z - z_2) = 0$$

Let A and B are the exterminties of diameter having position vector $\vec{a}$ and $\vec{b}$ repectively. Then equation of sphere is $(\vec{r} - \vec{a})(\vec{r} - \vec{b}) = 0$, where $\vec{r}$ is the position vector of any general point on the sphere.

Conditions of Tangency of a Plane to a Sphere

If plane ax + by + cz + d = 0 touches a sphere

$$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$$

then distance of plane from centre of sphere is equal to radius of sphere.

$$\therefore \quad \left|\frac{-au - bv - cw + d}{\sqrt{a^2 + b^2 + c^2}}\right| = \sqrt{u^2 + v^2 + w^2 - k}$$

Also, condition for the plane $\vec{r} \cdot \vec{n} = d$ to touch the sphere $|\vec{r} - \vec{c}| = a$ is $\frac{|\vec{c} \cdot \vec{n} - d|}{|\vec{n}|} = a$.

Important Facts Related to Sphere

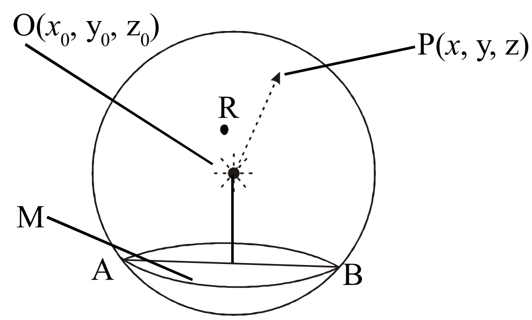
1. The condition of orthogona intersection of two sphere is

$$2u_1u_2 = 2v_1v_2 + 2w_1w_2 = d_1 + d_2$$

2. The section of a sphere cut by a plane through its centre is a circle called the greater circle.
3. If sphere S_1 and S_2 touch each other then $S_1 - S_2 = 0$ is common tangent plane. If S_1 and S_2 intersect each other, then $S_1 - S_2 = 0$ is equation of plane of common circle.
4. If a sphere is intersected by a plane, then the locus of set of points common to bote sphere and plane is always a circle.

   If length of perpendicular from the centre of to the plane is P, then radius of circle,

$$MB = \sqrt{R^2 - p^2}$$



If p = R, then plane touches the sphere and, if p > R, then the plane does not touch the sphere.

✷ Area of Triangle

Let $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$be the vertices of ΔABC, then

Area of triangle = $\frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$

$$= \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix}$$

✷ Volume of Tetrahedron

Let $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$, $C(x_3, y_3, z_3)$ and $D(x_4, y_4, z_4)$ be the vertices of tetrahedron, then volume of tetrehedron is given by

$$V = \frac{1}{6}\begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_4 & y_4 & z_4 & 1 \end{vmatrix}$$

Note :

- The distance of a point P(x, y, z) from the origin is $\sqrt{x^2+y^2+z^2}$.
- The distance of a point P(x, y, z) from x-axis is $\sqrt{y^2+z^2}$.
- If R is the mid point of PQ, then coordinates of R are $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$.
- The Direction cosines of a line are definded as the direction cosines of any directed line segment on it.
- Direction ratios of a line are not unique.
- The direction ratios of a line joining the points be $x_2 - x_1$, $y_2 - y_1$ and $z_2 - z_1$ and direction cosines are $\lambda(x_2 - x_1)$, $\lambda(y_2 - y_1)$ and $\lambda(z_2 - z_1)$.
- If $\vec{r} = a\hat{i} + b\hat{j} + c\hat{k}$ be a vector having direction cosine l, m, n. Then, $l = \frac{a}{|\vec{r}|}$, $m = \frac{b}{|\vec{r}|}$, $n = \frac{c}{|\vec{r}|}$
- If direction ratios of $\vec{r}$ are a, b, c then

$$\vec{r} = \frac{|\vec{r}|}{\sqrt{a^2+b^2+c^2}}(a\hat{i} + b\hat{j} + c\hat{k})$$

- Any vector equally inclined to all the three axes have direction cosines as $\left(\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}\right)$
- The vector equation of a plane passing through a point having position vector $\vec{a}$ and normal to vector $\vec{n}$ is $(\vec{r} - \vec{a})\cdot\vec{n} = 0$.
- Equation of a plane which passes through the point (x_1, y_1, z_1) and parallel to two lines whose DR's are $(\alpha_1, \beta_1, \gamma_1)$ and $(\alpha_2, \beta_2, \gamma_2)$ is $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ \alpha_1 & \beta_1 & \gamma_1 \\ \alpha_2 & \beta_2 & \gamma_2 \end{vmatrix} = 0$
- Intersection points of two planes also form a straight line, which is known as line of intersection bu this is an unsymmetrical form ot eh straight line.
- The line $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ lies in the plane $a_2x + b_2y + c_2z + d_2 = 0$ then

then $a_2x_1 + b_2y_1 + c_2z_1 + d_1 = 0$

and $a_1a_2 + b_1b_2 + c_1c_2 = 0$